Electric Potential

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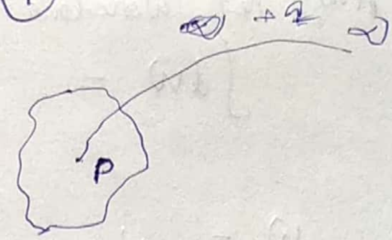


Electrostatic Potential :-

Workdone to bring a unit positive charge from infinity to any point in the electric field is called electric potential.

OR. The workdone in moving a unit positive charge from infinity to that point against the electrostatic force of the electric field of charge irrespective of the path followed.

Now, potential $(V) = \frac{W}{q}$; if $q = 1\,C$, then $\boxed{V = W}$

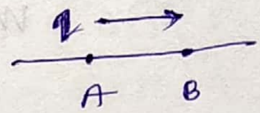
unit : Potential (V) $= \frac{1J}{1C} = 1V$ unit of potential = volt.

Note $V_P - V_\infty = \frac{W}{q}$ [difference bet^n ∞ and P] here, $V_\infty \to 0$]

$V_\infty \to 0$

$\therefore \boxed{V_P = W/q}$ $\because$ Electric potential is a scalar quantity.

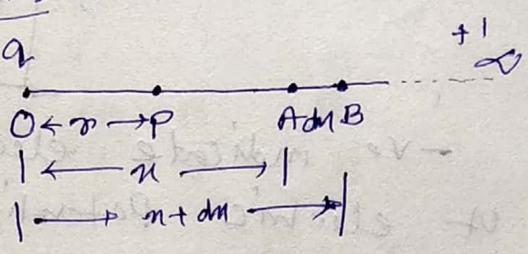
Potential Difference :-



$V_A - V_B = \Delta V = \frac{W_{AB}}{q}$

If $q = 1$, $\boxed{\Delta V = W_{AB}}$

Electric Potential due to a point charge :-



Let consider a point charge placed at O. we want to calculate potential at P at a distance r from O. Let us take elementary portion A and B at a distance x and x+dx from O.

Now, electrostatic force of repulsion bet^n q and +1 charge at at distance x.

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q \times 1}{x^2} \quad \text{---(i)}$$

Small workdone to displace charge +1 through dx.

$$dW = \vec{F} \cdot \vec{dx} = F\,dx \cos 180^\circ = -F\,dx \quad \text{---(ii)}$$

from eq^n (i) & (ii) we get

$$dW = -\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{x^2}\,dx \quad \text{---(iii)}$$

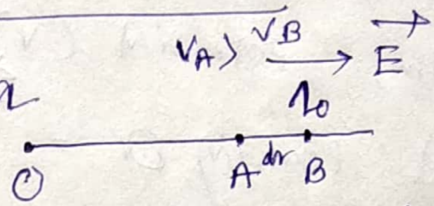
Now, net workdone from ∞ to P.

$$\int dw = \int_{x=\infty}^{x=r} -\frac{1}{4\pi\epsilon_0}\cdot\frac{q}{x^2}\,dx$$

$$W = V = -\frac{q}{4\pi\epsilon_0}\cdot\left[-\frac{1}{x}\right]_{\infty}^{r} = \frac{q}{4\pi\epsilon_0 r}$$

$$\boxed{\therefore\ V = \frac{q}{4\pi\epsilon_0 r}}$$

Relation betn electric field and potential



Force acting on point B due charge q_0

$\vec{F} = q_0 E$ —— (i)

Now, workdone to move charge q_0 from B to A through ~~dis~~ dx.

$$dw = \vec{F}\cdot\vec{dx} = F\,dx\cos 180^\circ = -F\,dx \quad \text{(ii)}$$

from eqn (i) & (ii)

$$dw = -q_0 E\,dx$$

or, $$\frac{dw}{q_0} = -E\,dx$$

or, $$dV = -E\,dr \qquad \left[dV = \frac{dw}{q_0}\right]$$

or, $$\boxed{E = -\frac{dV}{dr}}$$

'-ve' indicate electric field intensity is the direction of electric potential decrease.

OR. electric potential increase to the opposite direction of electric field intensity.

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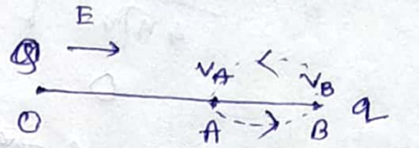
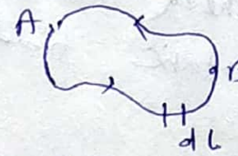
Electrostatic force is Conservative :



For conservative force;

(i) Workdone by the ~~any~~ force depends on initial and final position does not depends on path.



(ii) Work by the closed ~~surface~~ path is zero.

Workdone in moving charge q from B to A ; $W_{BA} = q_0 (V_A - V_B)$ — (1)

" " " " from A to B , $W_{AB} = q_0 (V_B - V_A)$ — (2)

Now from eq^n (1) & (2) $W_{BA} + W_{AB} = q_0 [V_A - V_B + V_B - V_A] = 0$

Hence, for closed line integral $\boxed{\oint \vec{E} \cdot d\vec{l} = W_{BA} + W_{AB} = 0} \rightarrow (3)$

According to stoke's law, $\boxed{\vec{\nabla} \times \vec{E} = 0}$ — (4)

Hence electrostatic field is conservative.

Poisson's and Laplace equation

We have differential form of Gauss theorem, $\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$ — (1)

Again we know, $\vec{E} = - \vec{\nabla} V$ — (ii)

Now, from eq^n (1) & (2). $\vec{\nabla} \cdot (- \vec{\nabla} V) = \rho / \epsilon_0$

$$\boxed{\Rightarrow \nabla^2 V = - \rho / \epsilon_0}$$ — poisson's eq^n

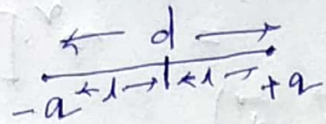
Now, for field free region $\rho = 0$

$$\boxed{\therefore \nabla^2 V = 0}$$ — This is Laplace's equation.

Electric Dipole

Electric dipole or bipole :-

Two equal and opposite charge separated by a small distance is called electric dipole.

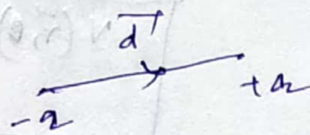


As $l = d/2$

$\underline{d = 2l}$

Electric dipole moment :-

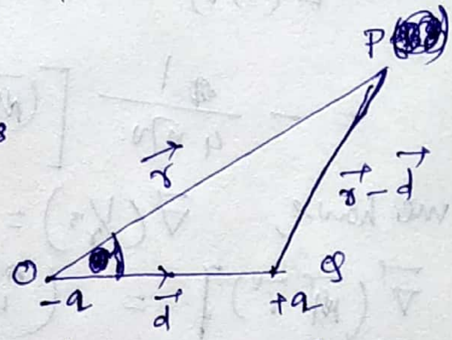
The product of magnitude of one charge and distance bet^n them. It is denoted by $\vec{P}$ or $\vec{M}$ and it's vector quantity and directed from negative charge to positive charge.



$$\boxed{\vec{P} \text{ or } \vec{M} = q\vec{d}}$$

unit = C-m.

Obtain expression for potential and field due to an electric dipole:



Let consider an electric dipole having charges q & -q separated by a distance d. We want to calculate potential and field at P.

Let us take the origin O of the co-ordinated system at the location of -q.

Potential at P due to $-q = -\dfrac{q}{4\pi\epsilon_0|\vec{r}|}$

" " " $+q = \dfrac{q}{4\pi\epsilon_0|\vec{r}-\vec{d}|}$

Total potential at P due to dipole.

$$V(\vec{r}) = \left[\frac{q}{4\pi\epsilon_0|\vec{r}-\vec{d}|} - \frac{q}{4\pi\epsilon_0|\vec{r}|}\right]$$

$$V(\vec{r}) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{|\vec{r}-\vec{d}|} - \frac{1}{|\vec{r}|}\right] \quad \text{--- (1)}$$

⑧

Now, $|\vec{r}-\vec{d}|^2 = |\vec{r}-\vec{d}|\cdot|\vec{r}-\vec{d}| = \vec{r}\cdot\vec{r} - \vec{r}\cdot\vec{d} - \vec{d}\cdot\vec{r} + \vec{d}\cdot\vec{d}$

$$= r^2 + d^2 - 2rd\cos\theta = r^2\left[1 + \underbrace{\frac{d^2}{r^2}}_{0} - \frac{2d\cos\theta}{r}\right]$$

$$= r^2\left[1 - \frac{2d\cos\theta}{r}\right]$$

$\frac{d}{r} \to$ small

$\frac{d^2}{r^2} \to 0$

$$|\vec{r}-\vec{d}| = r\left[1 - \frac{2d\cos\theta}{r}\right]^{1/2}$$

$$\frac{1}{|\vec{r}-\vec{d}|} = \frac{1}{r}\left[1 + \frac{d\cos\theta}{r}\right] = \frac{1}{r}\left[1 + \frac{dr\cos\theta}{r^2}\right]$$

$$= \frac{1}{r} + \frac{\vec{d}\cdot\vec{r}}{r^3}$$

Again from eqn ① $v(r,\theta) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} + \frac{\vec{d}\cdot\vec{r}}{r^3} - \frac{1}{r}\right]$

$$\boxed{V(\vec{r},\theta) = \frac{q}{4\pi\epsilon_0}\frac{\vec{d}\cdot\vec{r}}{r^3}}$$

Again, $\vec{\nabla}\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$ $V(\vec{r},\theta) = -\frac{\vec{M}\cdot\vec{\nabla}(1/r)}{4\pi\epsilon_0}$ $\vec{M} = q\vec{d}$

Now, $E(\vec{r},\theta) = -\vec{\nabla}V(\vec{r}) = -\vec{\nabla}\left[\frac{1}{4\pi\epsilon_0}\frac{\vec{M}\cdot\vec{r}}{r^3}\right]$

$$= -\frac{1}{4\pi\epsilon_0}\left[(\vec{M}\cdot\vec{r})\vec{\nabla}(1/r^3) + \frac{1}{r^3}\{\vec{\nabla}(\vec{M}\cdot\vec{r})\}\right]$$

we have, $\vec{\nabla}(1/r^3) = -\frac{3\vec{r}}{r^5}$

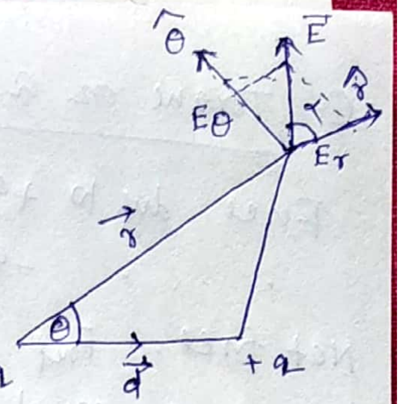
$$\vec{\nabla}(\vec{M}\cdot\vec{r})] = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(\hat{i}M_x + \hat{j}M_y + \hat{k}M_z\right)\left(\hat{i}x + \hat{j}y + \hat{k}z\right)$$

$$= \hat{i}M_x + \hat{j}M_y + \hat{k}M_z = \vec{M}$$

$$E(\vec{r},\theta) = -\frac{1}{4\pi\epsilon_0}\left[(\vec{M}\cdot\vec{r})\left(-\frac{3\vec{r}}{r^5}\right) + \frac{1}{r^3}\vec{M}\right]$$

$$\boxed{E(\vec{r},\theta) = \frac{1}{4\pi\epsilon_0}\left[\frac{3(\vec{M}\cdot\vec{r})\vec{r}}{r^5} - \frac{\vec{M}}{r^3}\right]}$$

We have $E(r,\theta) = -\vec{\nabla} V$ (9)



Two dimensional polar co-ordinate

$$E(r,\theta) = -\left[\frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{\theta}\right]\left(\frac{M\cos\theta}{4\pi\epsilon_0 r^2}\right)$$

$$= -\frac{M}{4\pi\epsilon_0}\left[-\frac{2\cos\theta}{r^3}\hat{r} - \frac{\sin\theta}{r^3}\hat{\theta}\right]$$

$$E(r,\theta) = +\frac{M}{4\pi\epsilon_0 r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$$

$$V(r,\theta) = \frac{M\cos\theta}{4\pi\epsilon_0 r^2}$$

$$E_r\hat{r} + E_\theta\hat{\theta} = \frac{M}{4\pi\epsilon_0 r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$$

$$= \frac{2M\cos\theta}{4\pi\epsilon_0 r^3}\hat{r} + \frac{M\sin\theta}{4\pi\epsilon_0 r^3}\hat{\theta}$$

$\therefore$ Radial Component of electric field $E_r = \dfrac{2M\cos\theta}{4\pi\epsilon_0 r^3}$

Angular " " " $E_\theta = \dfrac{M\sin\theta}{4\pi\epsilon_0 r^3}$

$$|\vec{E}| = \sqrt{(E_r)^2 + (E_\theta)^2} = \sqrt{\left(\frac{2M\cos\theta}{4\pi\epsilon_0 r^3}\right)^2 + \left(\frac{M\sin\theta}{4\pi\epsilon_0 r^3}\right)^2}$$

$$= \frac{M}{4\pi\epsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$\boxed{|\vec{E}| = \frac{M}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta + 1}}$$

$$\tan\alpha = \frac{E_\theta}{E_r} = \frac{M\sin\theta}{4\pi\epsilon_0 r^3} \times \frac{4\pi\epsilon_0 r^3}{2M\cos\theta} = \frac{1}{2}\tan\theta$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{1}{2}\tan\theta\right)}$$

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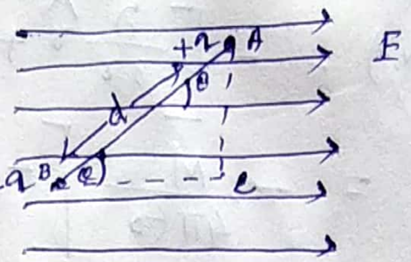
Torque on a dipole in uniform electric field

Force due to $+q$: $\vec{F_1} = q\vec{E}$ —(1)

" " $-q$, $\vec{F_2} = -q\vec{E}$ —(2)



Net force, $F_{net} = 0$

But two equal and opposite force constitute a couple. Due to this couple tends rotate the electric dipole clockwise and try to align to the electric field.

Now torque acting on the dipole = magnitude of the either force × perp^{r} distance bet^{n} two forces.

$= qE \times AC$

from ΔABC, $\sin\theta = \frac{AC}{AB}$ $\Rightarrow AC = d\sin\theta$

$= qE \times d\sin\theta = ME\sin\theta$

$$\boxed{\vec{\tau} = \vec{M} \times \vec{E}}$$

(iii) uniqueness theorem :-

two solution of Laplace's equation obeying same boundary condition differ by a const :

i.e. $\phi_1 - \phi_2 = C$

Let ϕ_1 & ϕ_2 two sol^n of Laplace's eq^n obeying boundary condition

(i) Dirichlet condition, i.e. $\phi_1 = \phi_2$

(ii) Neumann " i.e. $\frac{\partial \phi_1}{\partial n} = \frac{\partial \phi_2}{\partial n}$

Lt, $\phi = \phi_1 - \phi_2$

$\nabla^2 \phi = \nabla^2 \phi_1 - \nabla^2 \phi_2 = 0$

i.e. $\nabla^2 \phi = 0$

$\nabla^2 \phi_1 - \nabla^2 \phi_2 = 0$

$\phi_1 = \phi_2$